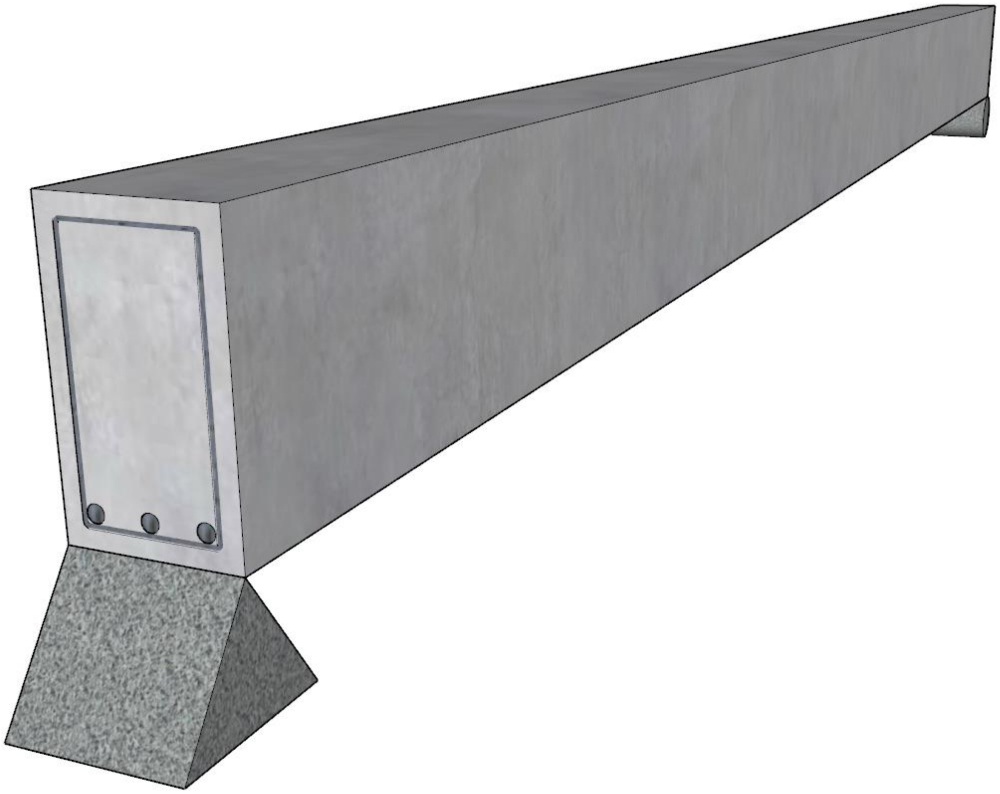
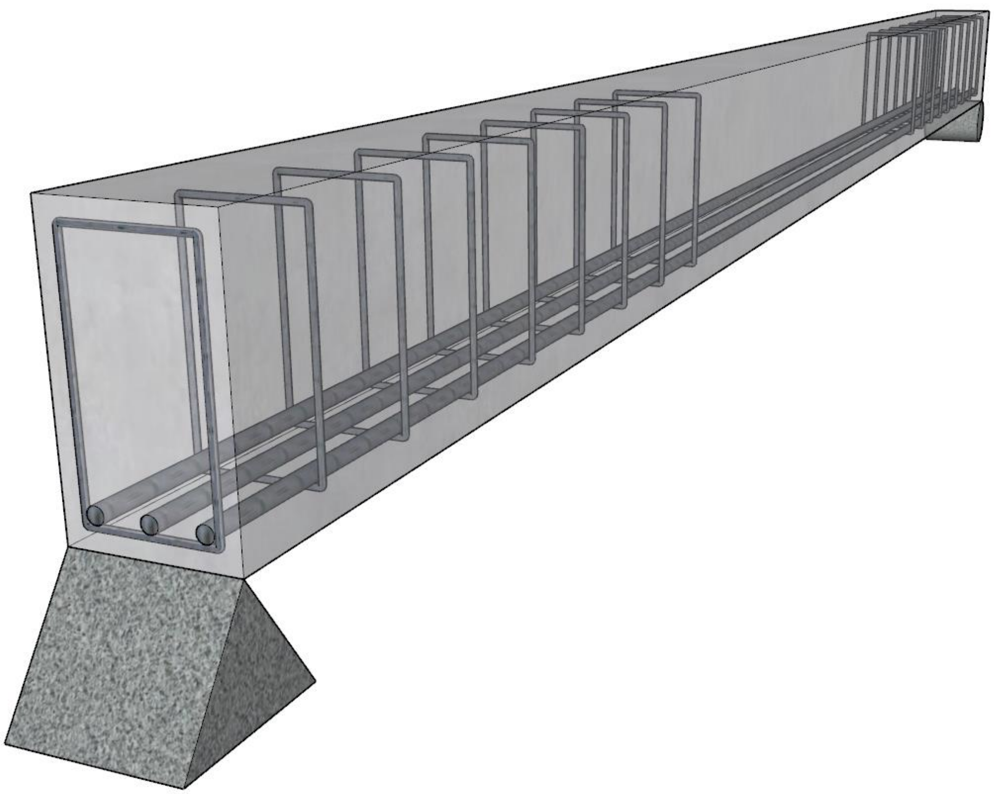
Simply Supported Reinforced Concrete Beam Analysis and Design (ACI 318-14)





Simply Supported Reinforced Concrete Beam Analysis and Design (ACI 318-14)

Simply supported beams consist of one span with one support at each end, one is a pinned support and the other is a roller support. The ends of these beams are free to rotate and have no moment resistance. There are numerous typical and practical applications of simply supported beams in buildings, bridges, industrial and special structures.

This example will demonstrate the analysis and design of the rectangular simply supported reinforced concrete beam shown below using ACI 318-14 provisions. Steps of the structural analysis, flexural design, shear design, and deflection checks will be presented. The results of hand calculations are then compared with the reference results and numerical analysis results obtained from the spBeam engineering software program by StructurePoint.

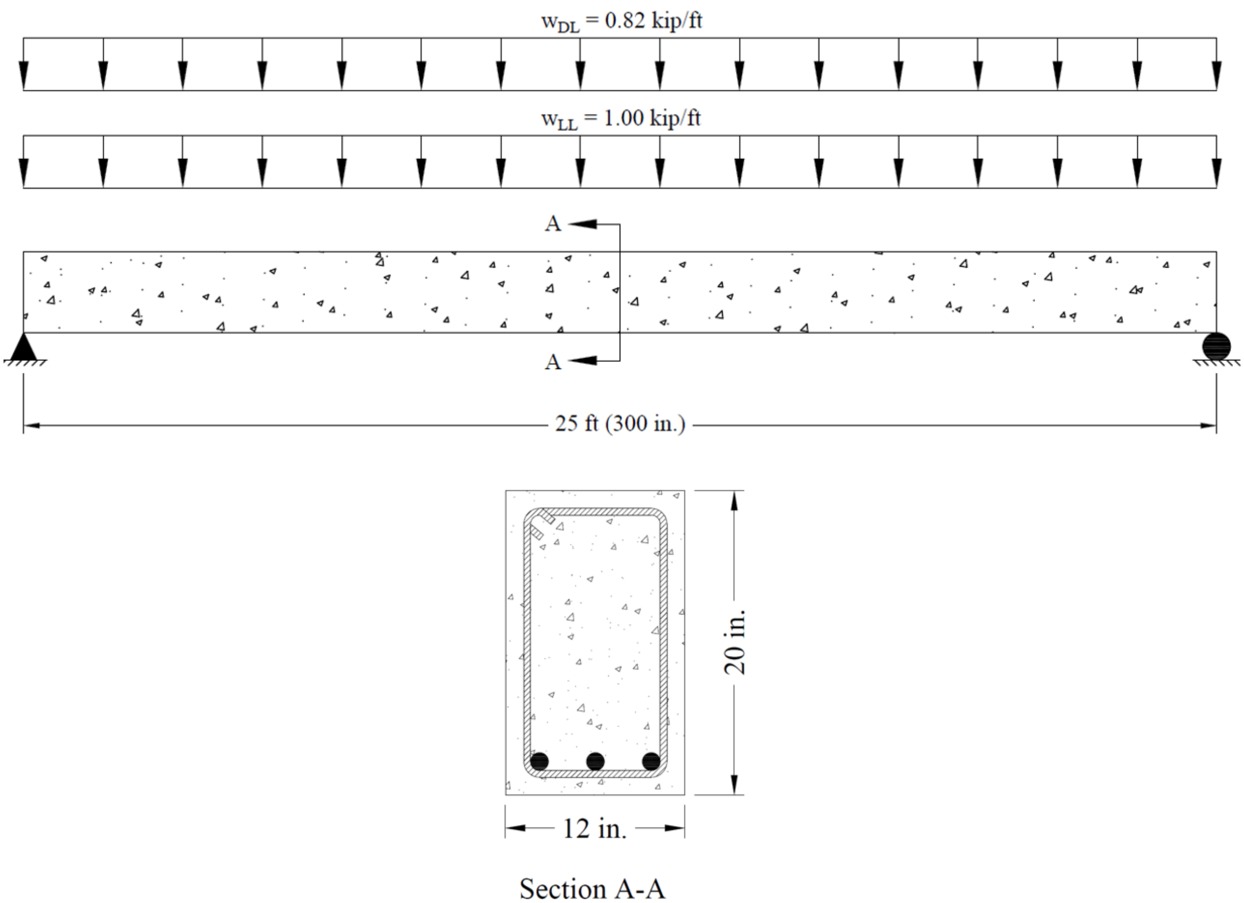


Figure 1 – Rectangular Simply Supported Reinforced Concrete Beam

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

References

Reinforced Concrete Structures, 2nd Edition, 2018, Omar Chaallal, Presses de l'Université du Québec.

spBeam Engineering Software Program Manual v5.00, STRUCTUREPOINT, 2015

Design Data

f_c' = 4.35 ksi normal weight concrete (w_c = 150 lb/ft^3)

f_y = 60 ksi

Uniform dead load, DL = 0.82 kip/ft (Reference neglected self-weight)

Uniform Live load, LL = 1.00 kip/ft

Beam span length, L = 25 ft

Use #9 bars for longitudinal reinforcement (A_s = 1.00 in.^{2}, d_b = 1.128 in.)

Use #3 bars for stirrups (A_s = 0.11 in.^{2}, d_b = 0.375 in.)

Clear cover = 1.5 in. ***ACI 318-14 (Table 20.6.1.3.1)***

a_{max} = maximum aggregate size = 0.75 in.

Solution

1. Preliminary Member Sizing

Check the minimum beam depth requirement of ***ACI 318-14 (Table 9.3.1.1)*** to waive deflection computations. Using the minimum depth for non-prestressed beams in ***Table 9.3.1.1***.

$$h_{\min} = \frac{l_n}{16} = \frac{300 \text{ in.}}{16} = 18.75 \text{ in. (For simply supported beams)}$$ ***ACI 318-14 (Table 9.3.1.1)***

Therefore, since h_{min} = 18.75 in. < h = 20 in. the preliminary beam depth satisfies the minimum depth requirement, and the beam deflection computations are not required.

In absence of initial dimensions, the width of the rectangular section (b) may be chosen in the following range recommended by the reference:

$$\left(\frac{1}{2} \times h = 10 \text{ in.}\right) \le b = 12 \text{ in.} \le \left(\frac{2}{3} \times h = 13.33 \text{ in.}\right)$$ ***o.k.***

2. Load and Load combination

For the factored Load

$$w_u = 1.2 \times DL + 1.6 \times LL$$ ***ACI 318-14 (Eq. 5.3.1b)***

$$w_u = 1.2 \times 0.82 + 1.5 \times 1.00 = 2.58 \text{ kip/ft}$$

3. Structural Analysis

Simply supported beams can be analyzed by calculating shear and moment diagrams or using Design Aid tables as shown below:

Shear and Moment Diagrams:

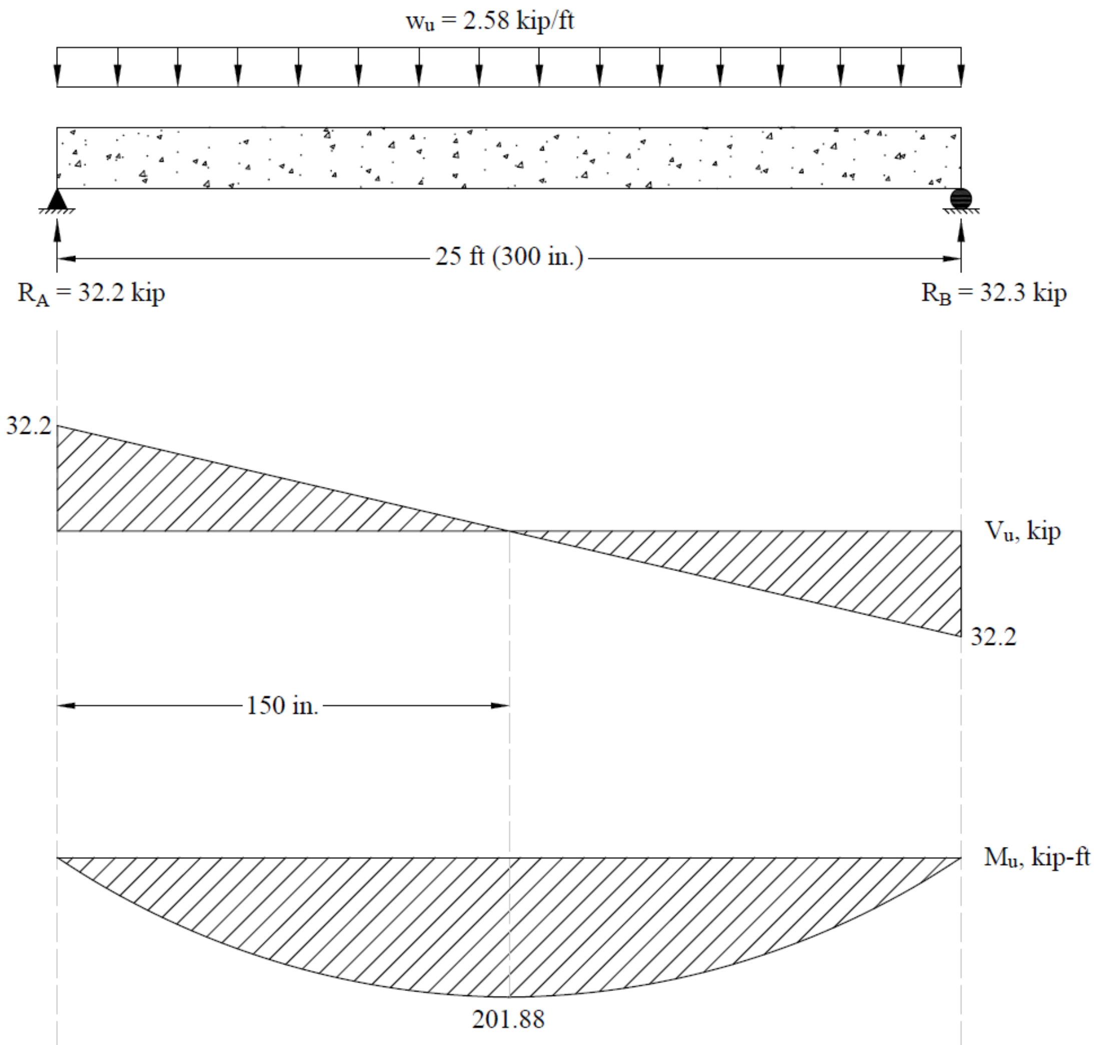


Figure 2 – Shear and Bending Moment Diagrams

Using Design Aid Tables:

$$V_u = R_A = R_B = \frac{w_u \times L}{2} = \frac{2.58 \times 25}{2} = 32.3 \text{ kip}$$

$$M_u = \frac{w_u \times L^2}{8} = \frac{2.58 \times 25^2}{8} = 201.88 \text{ kip-ft}$$

SIMPLE BEAM – UNIFORMLY DISTRIBUTED LOAD

$R = V$ .... $= \dfrac{w\ell}{2}$

V_x .... $= w\left(\dfrac{\ell}{2} - x\right)$

M_{max} (at center) .... $= \dfrac{w\ell^2}{8}$

M_x .... $= \dfrac{wx}{2}(\ell - x)$

Δ_{max} (at center) .... $= \dfrac{5w\ell^4}{384EI}$

Δ_x .... $= \dfrac{wx}{24EI}(\ell^3 - 2\ell x^2 + x^3)$

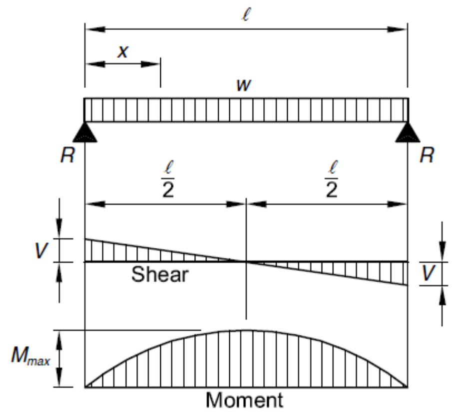


Figure 3 – Design Aid Tables (Beam Design Equations and Diagrams) – PCI Design Handbook

4. Flexural Design

4.1. Required and Provided Reinforcement

For this beam, the moment at the midspan governs the design as shown in the previous Figure.

$$M_u = 201.88 \text{ kip-ft}$$

Use #9 bars with 1.5 in. concrete clear cover per ***ACI 318-14 (Table 20.6.1.3.1)***. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, *d*, is calculated below:

$$d = h - \left(\text{clear cover} + d_{b,stirrups} + \frac{d_{Longitudinal\ bar}}{2}\right)$$

$$d = 20 - \left(1.50 + 0.375 + \frac{1.128}{2}\right) = 17.56 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the beam section (*jd*). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and *jd* will be taken equal to *0.889d*. The assumptions will be verified once the area of steel is finalized.

$$jd = 0.889 \times d = 0.889 \times 17.56 = 15.62 \text{ in.}$$

$b = 12\text{ in.}$

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{201.88 \times 12{,}000}{0.9 \times 60{,}000 \times 15.62} = 2.872\text{ in.}^2$$

Recalculate 'a' for the actual $A_s = 2.872\text{ in.}^2$: $a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{2.872 \times 60{,}000}{0.85 \times 4{,}350 \times 12} = 3.88\text{ in.}$

$$c = \frac{a}{\beta_1} = \frac{3.88}{0.83} = 4.67\text{ in.}$$

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000}$$ ***ACI 318-14 (Table 22.2.2.4.3)***

$$\beta_1 = 0.85 - \frac{0.05 \times (4350 - 4000)}{1000} = 0.83$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) \times d_t - 0.003 = \left(\frac{0.003}{4.67}\right) \times 17.56 - 0.003 = 0.0083 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left(d - \frac{a}{2}\right)} = \frac{201.88 \times 12{,}000}{0.9 \times 60{,}000 \times \left(17.56 - \frac{3.88}{2}\right)} = 2.872\text{ in.}^2$$

The minimum reinforcement shall not be less than

$$A_{s,\min} = \frac{3 \times \sqrt{f'_c}}{f_y} \times b_w \times d = \frac{3\sqrt{4350}}{60000} \times 12 \times 17.56 = 0.695\text{ in.}^2$$ ***ACI 318-14 (9.6.1.2(a))***

And not less than

$$A_{s,\min} = \frac{200}{f_y} \times b_w \times d = \frac{200}{60000} \times 12 \times 17.56 = 0.702\text{ in.}^2$$ ***ACI 318-14 (9.6.1.2(b))***

$$\therefore A_{s,\min} = 0.702\text{ in.}^2$$

$$A_{s,req} = \max\begin{Bmatrix} A_s \\ A_{s,\min} \end{Bmatrix} = \max\begin{Bmatrix} 2.872 \\ 0.702 \end{Bmatrix} = 2.872\text{ in.}^2$$

Provide 3 – #9 bars:

$$A_{s,prov} = 3 \times 1.00 = 3.00\text{ in.}^2 > A_{s,req} = 2.872\text{ in.}^2$$

4.2. Spacing of Longitudinal Reinforcement

$$s_{provided}=\frac{(b-2\times d_s)}{\#of\ \ bars-1}=\frac{\left(12-2\times\left(2\frac{5}{8}\right)\right)}{3-1}=3.38\text{ in.}$$

Where d_s = 2.625 in. for #3 stirrup as shown in the following Figure. ***CRSI 2002 (Figure 12-9)***

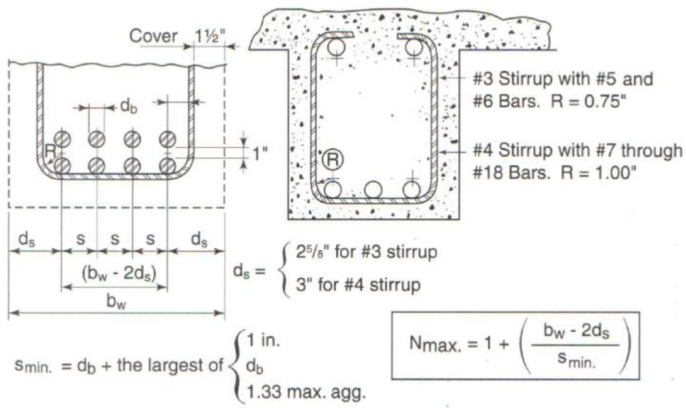


Figure 4 - Maximum number of bars in beams

The maximum allowed spacing (s_{max}):

$$s_{max}=15\left(\frac{40000}{f_s}\right)-2.5c_c\le 12\left(\frac{40000}{f_s}\right)$$ ***ACI 318-14 (Table 24.3.2)***

$c_c=1.5$ in.

Use $f_s=\frac{2}{3}f_y=40000$ psi ***ACI 318-14 (24.3.2.1)***

$$s_{max}=\min\begin{Bmatrix}15\times\left(\frac{40000}{40000}\right)-2.5\times1.5\\12\times\left(\frac{40000}{40000}\right)\end{Bmatrix}=\min\begin{Bmatrix}10.31\\12\end{Bmatrix}=10.31\text{ in.}$$

The minimum allowed spacing (s_{min}):

$$s_{min}=d_b+\max\begin{Bmatrix}1\\d_b\\1.33\times\max.agg.\end{Bmatrix}$$ ***CRSI 2002 (Figure 12-9)***

Where the maximum aggregate size is ¾”

$$s_{min}=1.00+\max\begin{Bmatrix}1.00\\1.128\\1.33\times0.75=1.00\end{Bmatrix}=1.00+1.128=2.26\text{ in.}$$

s_{min} = 2.26 in. < $s_{provided}$ = 3.38 in. < s_{max} = 10.31 in.

Therefore, 3 - #9 bars are ***o.k.***

5. Shear Design

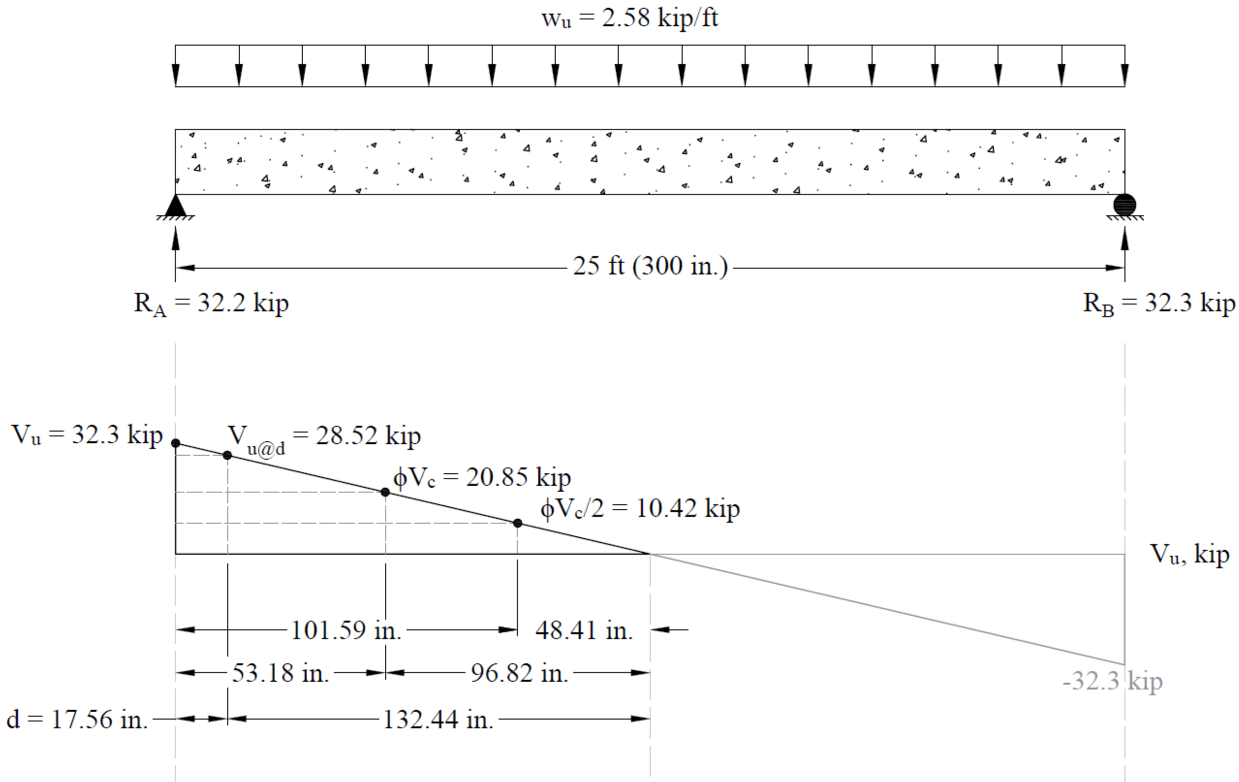


Figure 5 – Shear Diagram for Simply Supported Beam

$$V_u = 32.3 \text{ kips}$$

$$V_{u@d} = 32.3 \times \frac{150-17.56}{150} = 28.52 \text{ kips}$$

Shear strength provided by concrete

$$\phi V_c = \phi \times 2 \times \sqrt{f_c^{'}} \times b_w \times d$$ **_ACI 318-14 (Eq. 22.5.5.1)_**

$$\phi V_c = 0.75 \times 2 \times \sqrt{4350} \times 12 \times 17.56 = 20.85 \text{ kips}$$

$$\frac{\phi V_c}{2} = 10.42 \text{ kips} < V_u = 32.3 \text{ kips}$$

Since $V_u > \phi V_c/2$, shear reinforcement is required.

Try # 3, Grade 60 two-leg stirrups (A_v = 2 x 0.11 = 0.22 in.^{2}).

The nominal shear strength required to be provided by shear reinforcement is

$$V_s = V_n - V_c = \frac{V_{u@d}}{\phi} - \frac{\phi V_c}{\phi} = \frac{28.52}{0.75} - \frac{20.85}{0.75} = 10.23 \text{ kips}$$

If V_s is greater than $8\sqrt{f_c'}b_w d$, then the cross-section has to be revised as ***ACI 318-14*** limits the shear capacity to be provided by stirrups to $8\sqrt{f_c'}b_w d$. ***ACI 318-14 (22.5.1.2)***

$$8\times\sqrt{f_c'}\times b_w\times d = 8\times\sqrt{4350}\times 12\times 17.56 = 111.19 \text{ kips} \rightarrow \therefore \text{ section is adequate}$$

$$\left(\frac{A_v}{s}\right)_{req} = \frac{V_{u@d}-\phi V_c}{\phi\times f_{yt}\times d} = \frac{28.52-20.85}{0.75\times 60\times 17.56} = 0.0097 \text{ in.}^2/\text{ in.}$$ ***ACI 318-14 (22.5.10.5.3)***

$$s_{req} = \frac{A_v}{\left(\frac{A_v}{s}\right)_{req}} = \frac{0.22}{0.0097} = 22.67 \text{ in.}$$

$$\left(\frac{A_v}{s}\right)_{\min} = \max\begin{Bmatrix}\dfrac{0.75\times\sqrt{f_c'}\times b_w}{f_{yt}}\\ \dfrac{50\times b_w}{f_{yt}}\end{Bmatrix}$$ ***ACI 318-14 (10.6.2.2)***

$$\left(\frac{A_v}{s}\right)_{\min} = \max\begin{Bmatrix}\dfrac{0.75\times\sqrt{4350}\times 12}{60000}\\ \dfrac{50\times 12}{60000}\end{Bmatrix} = \max\begin{Bmatrix}0.0099\\ 0.0100\end{Bmatrix} = 0.0100 \text{ in.}^2/\text{in.} > \left(\frac{A_v}{s}\right)_{req} = 0.0097 \text{ in.}^2/\text{ in.}$$

$$\therefore\left(\frac{A_v}{s}\right)_{req} = 0.0100 \text{ in.}^2/\text{ in.}$$

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per ***ACI 318-14 (9.7.6.2.2)***.

$$4\times\sqrt{f_c'}\times b_w\times d = 4\times\sqrt{4350}\times 12\times 17.56 = 55.59 \text{ kips} > V_s = 10.23 \text{ kips}$$

Therefore, maximum stirrup spacing shall be the smallest of *d/2* and *24 in.* ***ACI 318-14 (Table 9.7.6.2.2)***

$$s_{\max} = \min\begin{Bmatrix}d/2\\ 24 \text{ in.}\end{Bmatrix} = \min\begin{Bmatrix}17.56/2\\ 24 \text{ in.}\end{Bmatrix} = \min\begin{Bmatrix}8.78 \text{ in.}\\ 24 \text{ in.}\end{Bmatrix} = 8.78 \text{ in.}$$

This value governs over the required stirrup spacing of 22.67 in which was based on the demand.

Therefore, s_{max} value is governed by the spacing limit per ***ACI 318-14 (9.7.6.2.2)***, and is equal to 8.78 in.

Use # 3 @ 8.3 in. stirrups

$$\phi V_n = \frac{\phi\times A_v\times f_{yt}\times d}{s} + \phi V_c$$ ***ACI 318-14 (22.5.1.1 and 22.5.10.5.3)***

$$\phi V_n = \frac{0.75\times 0.22\times 60\times 17.56}{8.30} + 20.85 = 20.95 + 20.85 = 41.79 \text{ kips} > V_{u@d} = 28.52 \text{ kips}$$ ***o.k.***

Compute where $\frac{V_u}{\phi}$ is equal to $\frac{V_c}{2}$, and the stirrups can be stopped

$$x = \frac{\frac{V_u}{\phi} - \frac{V_c}{2}}{\frac{V_u}{\phi}} \times \frac{l}{2} = \frac{\frac{32.3}{0.75} - \frac{20.85}{0.75 \times 2}}{\frac{32.3}{0.75}} \times \frac{25 \times 12}{2} = 101.59 \text{ in.}$$

Use 16 - # 3 @ 8.30 in. o.c., Place 1st stirrup 3 in. from the face of the column.

6. Deflection Control (Serviceability Requirements)

Since the preliminary beam depth met minimum depth requirement, the deflection calculations are not required. However, the calculations of immediate and time-dependent deflections are covered in detail in this section for illustration and comparison with spBeam model results for simply supported beam.

6.1. Immediate (Instantaneous) Deflections

Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D+L_{Full}$) is used to obtain immediate deflections of the simply supported beam in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests.

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a.

Unless deflections are determined by a more comprehensive analysis, immediate deflection shall be computed using elastic deflection equations using I_e from Eq. (24.2.3.5a) at midspan for simple and continuous spans, and at the support for cantilevers. ***ACI 318-14 (24.2.3.7)***

The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the beam. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g$$ ***ACI 318-14 (Eq. 24.2.3.5a)***

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously (sustained live load = 0).

$$M_{DL} = M_{DL+LL\_sustained} = \frac{w_{DL} \times L^2}{8} = \frac{0.82 \times (25)^2}{8} = 64.06 \text{ kip-ft}$$

$$M_{DL+LL} = \frac{(w_{DL} + w_{LL}) \times L^2}{8} = \frac{(0.82 + 1.00) \times (25)^2}{8} = 142.19 \text{ kip-ft}$$

M_{cr} = cracking moment.

$$M_{cr} = \frac{f_r I_g}{Y_t} = \frac{(494.66) \times (8000)}{10} \times \frac{1}{12000} = 32.98 \text{ kip-ft}$$ ***ACI 318-14 (Eq. 24.2.3.5b)***

f_r = Modulus of rapture of concrete.

$$f_r = 7.5\lambda\sqrt{f'_c} = 7.5\times1.0\times\sqrt{4350} = 494.66\text{ psi}$$ ***ACI 318-14 (Eq. 19.2.3.1)***

I_g = Moment of inertia of the gross uncracked concrete section

$$I_g = \frac{b\times h^3}{12} = \frac{12\times20^3}{12} = 8000\text{ in.}^4$$

$$y_t = \frac{h}{2} = \frac{20}{2} = 10\text{ in.}$$

I_{cr} = moment of inertia of the cracked section transformed to concrete.

CAC Concrete Design Handbook 4th Edition (5.2.3)

The critical section at midspan is reinforced with 3 – #9 bars.

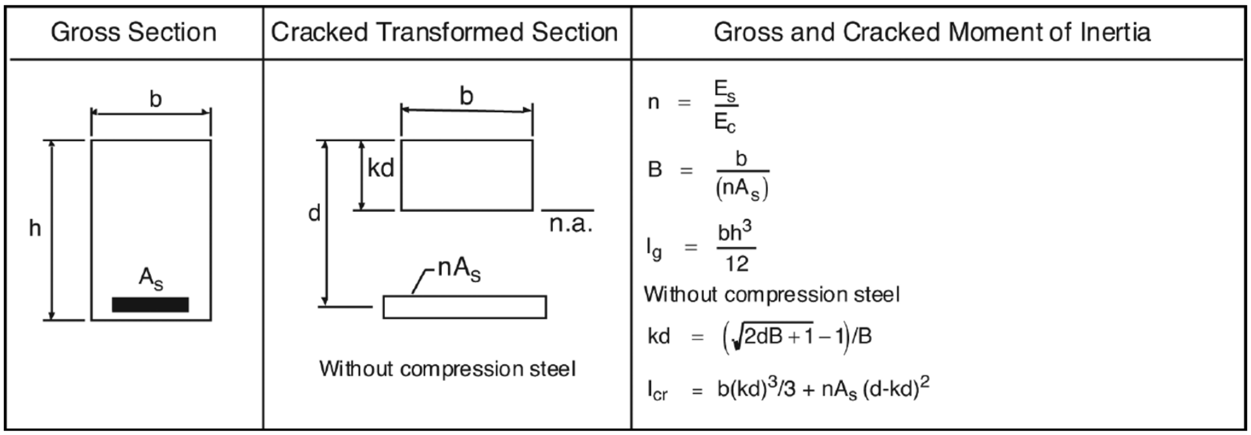


Figure 6 – Gross and Cracked Moment of Inertia of Rectangular Section (PCA Notes Table 10-2)

E_c = Modulus of elasticity of concrete.

$$E_c = w_c^{1.5}33\sqrt{f'_c} = 150^{1.5}\times33\times\sqrt{4350} = 3998.5\text{ ksi}$$ ***ACI 318-14 (19.2.2.1.a)***

$$n = \frac{E_s}{E_c} = \frac{29000}{3998.5} = 7.25$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$B = \frac{b}{n\,A_s} = \frac{12}{7.25\times(3\times1.00)} = 0.552\text{ in.}^{-1}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$kd = \frac{\sqrt{2dB+1}-1}{B} = \frac{\sqrt{2\times17.56\times0.552+1}-1}{0.552} = 6.37\text{ in.}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d-kd)^2$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{12\times6.37^3}{3} + 7.25\times(3\times1.00)\times(17.56-6.37)^2 = 3759\text{ in.}^4$$

For dead load service load level:

$$I_{ec} = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \text{ , since } M_{cr} = 32.98 \text{ kip-ft} < M_a = 64.06 \text{ kip-ft}$$ ***ACI 318-14 (24.2.3.5a)***

$$I_e = 3759 + \left(8000 - 3759\right)\left(\frac{32.98}{64.06}\right)^3 = 4337 \text{ in.}^4$$

The following Table provides a summary of the required parameters and calculated values needed for deflection calculation.

Table 1 – Effective Moment of Inertia Calculations (at midspan)								
I_g, in.^{4}	I_{cr}, in.^{4}	M_a, kip-ft			M_{cr}, kip-ft	I_e, in.^{4}		
		D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}
8000	3759	64.06	64.06	142.19	32.98	4337	4337	3812

After obtaining the effective moment of inertia, the maximum span deflection for the simply supported beam can be obtained from any available procedures or design aids (see Figure 3).

$$\Delta_{max} = \frac{5}{384} \times \frac{w \times L^4}{E_c \times I_e}$$

$$\Delta_{DL} = \frac{5}{384} \times \frac{820 \times 300^4}{\left(3998.48 \times 10^3\right) \times 4337} = 0.416 \text{ in.}$$

$$\Delta_{Total} = \frac{5}{384} \times \frac{\left(820 + 1000\right) \times 300^4}{\left(3998.48 \times 10^3\right) \times 3812} = 1.050 \text{ in.}$$

$$\Delta_{LL} = \Delta_{Total} - \Delta_{DL} = 1.050 - 0.416 = 0.634 \text{ in.} < \frac{L}{360} = \frac{300}{360} = 0.833 \text{ in. } \textbf{\textit{o.k.}}$$ ***ACI 318-14 (Table 24.2.2)***

6.2. Time-Dependent (Long-Term) Deflections (Δ_{lt})

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) are estimated as follows.

$$\Delta_{cs} = \lambda_{\Delta} \times (\Delta_{sust})_{Inst}$$ ***PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)***

The total time-dependent (long-term) deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1+\lambda_{\Delta}) + ((\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst})$$ ***CSA A23.3-04 (N9.8.2.5)***

Where:

$(\Delta_{sust})_{Inst} =$ Immediate (instantaneous) deflection due to sustained load, in.

$$\lambda_{\Delta} = \frac{\xi}{1+50\rho'}$$ ***ACI 318-14 (24.2.4.1.1)***

$(\Delta_{total})_{lt} =$ Time-dependent (long-term) total delfection, in.

$(\Delta_{total})_{Inst} =$ Total immediate (instantaneous) deflection, in.

For the exterior span

$\xi = 2$, consider the sustained load duration to be 60 months or more. ***ACI 318-14 (Table 24.2.4.1.3)***

$\rho' = 0$, conservatively.

$$\lambda_{\Delta} = \frac{2}{1+50\times 0} = 2$$

$$\Delta_{cs} = 2\times 0.416 = 0.831 \text{ in.}$$

$$\Delta_{cs} + \Delta_{LL} = 0.831 + 0.634 = 1.465 \text{ in.} > \frac{L}{240} = \frac{25\times 12}{240} = 1.25 \text{ in.}$$ ***(Exceeded)*** ***ACI 318-14 (Table 24.2.2)***

$$(\Delta_{total})_{lt} = 0.416\times(1+2)+(1.050-0.416) = 1.881 \text{ in.}$$

7. Simply Supported Beam Analysis and Design – spBeam Software

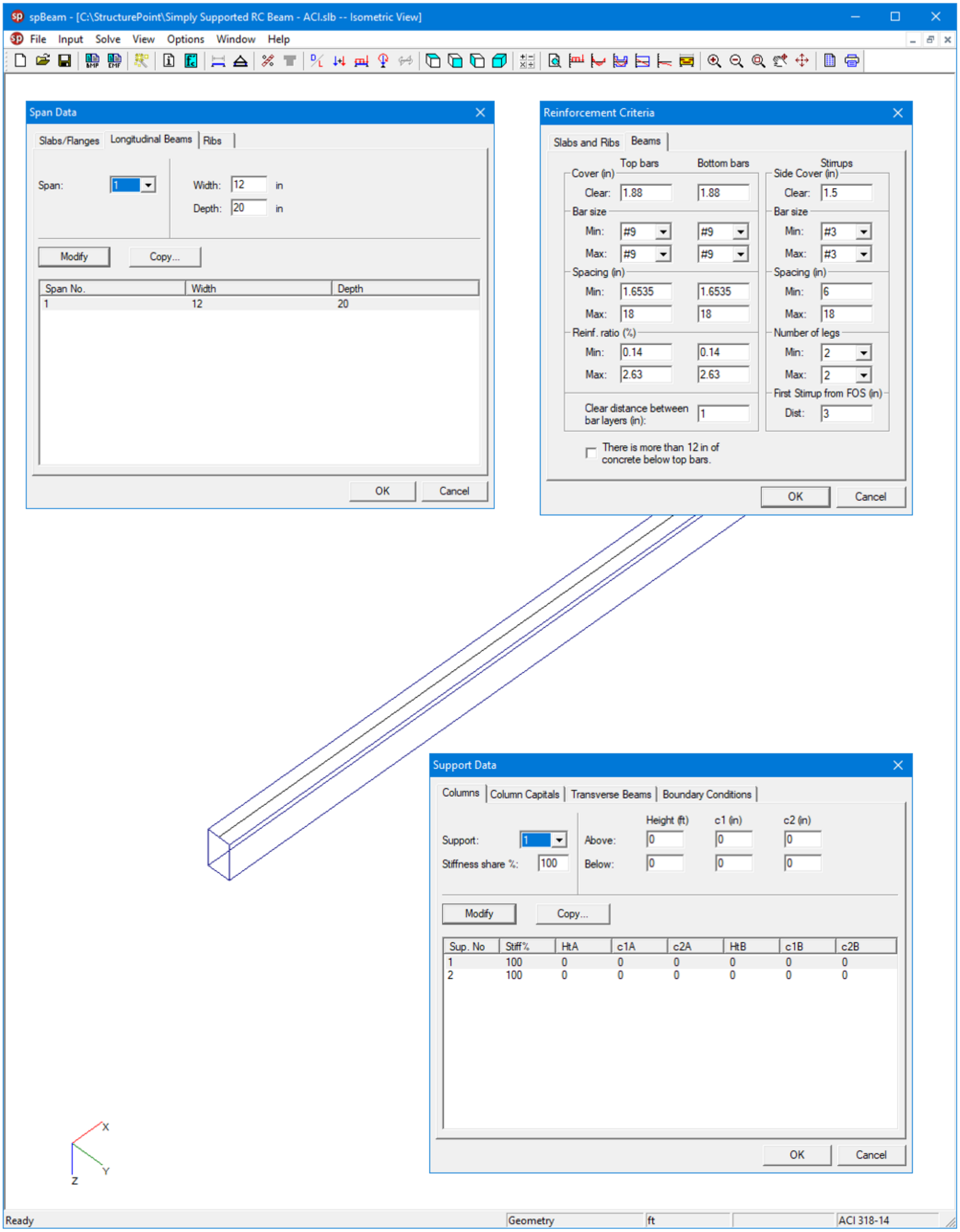
spBeam is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per latest American (ACI 318-14) and Canadian (CSA A23.3-14) codes. spBeam can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, spBeam is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

spBeam provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

Beam analysis and design requires engineering judgment in most situations to properly simulate the behavior of the targeted beam and take into account important design considerations such as: designing the beam as rectangular or T-shaped sections; using the effective flange width or the center-to-center distance between the beam and the adjacent beams. Regardless which of these options is selected, spBeam provide users with options and flexibility to:

1. Design the beam as a rectangular cross-section or a T-shaped section.
2. Use the effective or full beam flange width.
3. Include the flanges effects in the deflection calculations.
4. Invoke moment redistribution to lower negative moments
5. Using gross (uncracked) or effective (cracked) moment of inertia

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the simply supported beam discussed in this example.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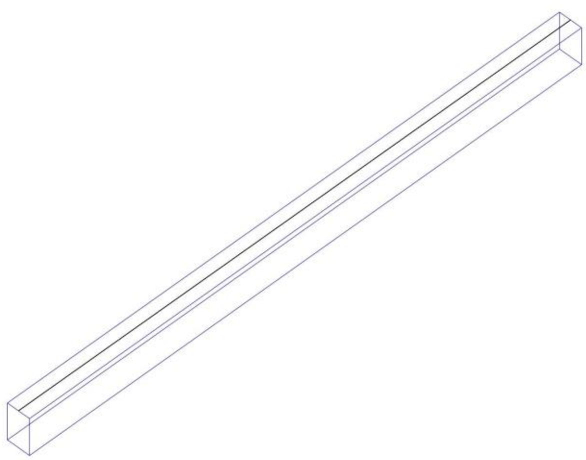
File Input Solve View Options Window Help
Span Data
Slabs/Flanges
Longitudinal Beams
Ribs
Span:
1
Width: 12 in
Depth: 20 in
Modify
Copy...
Span No. Width Depth
1 12 20
OK
Cancel
Reinforcement Criteria
Slabs and Ribs
Beams
Top bars
Bottom bars
Stirrups
Cover (in)
Clear: 1.88 1.88
Bar size
Min: #9 #9
Max: #9 #9
Spacing (in)
Min: 1.6535 1.6535
Max: 18 18
Reinf. ratio (%)
Min: 0.14 0.14
Max: 2.63 2.63
Clear distance between bar layers (in): 1
Side Cover (in)
Clear: 1.5
Bar size
Min: #3
Max: #3
Spacing (in)
Min: 6
Max: 18
Number of legs
Min: 2
Max: 2
First Stirrup from FOS (in)
Dist: 3
There is more than 12 in of concrete below top bars.
OK
Cancel
Support Data
Columns
Column Capitals
Transverse Beams
Boundary Conditions
Support: 1
Stiffness share %: 100
Height (ft) c1 (in) c2 (in)
Above: 0 0 0
Below: 0 0 0
Modify
Copy...
Sup. No Stiff% HtA c1A c2A HtB c1B c2B
1 100 0 0 0 0 0 0
2 100 0 0 0 0 0 0
OK
Cancel
X
Y
Z
Ready
Geometry
ft
ACI 318-14

spBeam v5.50
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams and One-way Slab Systems



Contents

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1. Input Echo

1.1. General Information

File Name	C:\Structur...\Simply Supported RC Beam - ACI.slb
Project	Simply Supported RC Beam
Frame	Simply Supported RC Beam
Engineer	SP
Code	ACI 318-14
Reinforcement Database	ASTM A615
Mode	Design
Number of supports =	2
Floor System	One-Way/Beam

1.2. Solve Options

Live load pattern ratio = 0%
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
Moment redistribution NOT selected.
Effective flange width calculations NOT selected.
Rigid beam-column joint NOT selected.
Torsion analysis and design NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	150	lb/ft^3
f'_c	4.35	ksi
E_c	3998.5	ksi
f_r	0.49466	ksi

1.3.2. Concrete: Columns

w_c	150	lb/ft^3
f'_c	4.35	ksi
E_c	3998.5	ksi
f_r	0.49466	ksi

1.3.3. Reinforcing Steel

f_y	60	ksi
f_{yt}	60	ksi
E_s	29000	ksi
Epoxy coated bars	No	

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in²	lb/ft		in	in²	lb/ft
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30
#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

1.5. Span Data

1.5.1. Slabs

Span	Loc	L1	t	wL	wR	H_{min}
		ft	in	ft	ft	in
1	Int	25.000	0.00	0.500	0.500	0.00

1.5.2. Ribs and Longitudinal Beams

Span	Ribs			Beams		Span
	b	h	Sp	b	h	H_{min}
	in	in	in	in	in	in
1	0.00	0.00	0.00	12.00	20.00	18.75

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	in	in	ft	in	in	ft	
1	0.00	0.00	0.000	0.00	0.00	0.000	100
2	0.00	0.00	0.000	0.00	0.00	0.000	100

1.6.2. Boundary Conditions

Support	Spring		Far End	
	K_z	K_{ry}	Above	Below
	kip/in	kip-in/rad		
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed

1.7. Load Data

1.7.1. Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.200	1.600

1.7.2. Line Loads

Case/Patt	Span	Wa	La	Wb	Lb
		lb/ft	ft	lb/ft	ft
Dead	1	820.00	0.000	820.00	25.000
Live	1	1000.00	0.000	1000.00	25.000

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#5	#8	#5	#8
Bar spacing	in	1.00	18.00	1.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	in	1.50		1.50	

There is NOT more than 12 in of concrete below top bars.

1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#9	#9	#9	#9	#3	#3
Bar spacing	in	1.65	18.00	1.65	18.00	6.00	18.00
Reinf ratio	%	0.14	2.63	0.14	2.63		
Clear Cover	in	1.88		1.88			
Layer dist.	in	1.00		1.00			
No. of legs						2	2
Side cover	in					1.50	
1st Stirrup	in					3.00	

There is NOT more than 12 in of concrete below top bars.

2. Design Results

2.1. Top Reinforcement

Span	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
		ft	k-ft	ft	in²	in²	in²	in	
1	Left	1.00	0.00	0.000	0.000	4.053	0.000	0.000	---
	Midspan	1.00	0.00	12.500	0.000	4.053	0.000	0.000	---
	Right	1.00	0.00	25.000	0.000	4.053	0.000	0.000	---

2.2. Top Bar Details

	Left				Continuous		Right			
Span	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
		ft		ft		ft		ft		ft
1	---		---		---		---		---	

2.3. Top Bar Development Lengths

	Left				Continuous		Right			
Span	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
		in		in		in		in		in
1	---		---		---		---		---	

2.4. Bottom Reinforcement

Span	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
	ft	k-ft	ft	in²	in²	in²	in	
1	1.00	201.88	12.500	0.702	4.053	2.873	3.507	3-#9

2.5. Bottom Bar Details

	Long Bars			Short Bars		
Span	Bars	Start	Length	Bars	Start	Length
		ft	ft		ft	ft
1	3-#9	0.00	25.00	---		

2.6. Bottom Bar Development Lengths

	Long Bars		Short Bars	
Span	Bars	DevLen	Bars	DevLen
		in		in
1	3-#9	47.42	---	

2.7. Flexural Capacity

		Top					Bottom				
Span	x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
	ft	in²	k-ft	k-ft			in²	k-ft	k-ft		
1	0.000	0.00	0.00	0.00	U1 All	OK	3.00	209.62	0.00	U1 All	OK
	8.750	0.00	0.00	0.00	U1 All	OK	3.00	209.62	183.71	U1 All	OK
	12.500	0.00	0.00	0.00	U1 All	OK	3.00	209.62	201.88	U1 All	OK
	16.250	0.00	0.00	0.00	U1 All	OK	3.00	209.62	183.71	U1 All	OK
	25.000	0.00	0.00	0.00	U1 All	OK	3.00	209.62	0.00	U1 All	OK

2.8. Longitudinal Beam Transverse Reinforcement Demand and Capacity

2.8.1. Section Properties

Span	d	$(A_v/s)_{min}$	ΦV_c
	in	in²/in	kip
1	17.56	0.0100	20.84

2.8.2. Beam Transverse Reinforcement Demand

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

			Required				Demand	
Span	Start	End	X_u	V_u	Comb/Patt	A_v/s	A_v/s	
	ft	ft	ft	kip		in²/in	in²/in	
1	0.250	4.616	1.463	28.52	U1/All	0.0097	0.0100	*8
	4.616	7.770	4.616	20.37	U1/All	0.0000	0.0100	*8
	7.770	10.923	7.770	12.22	U1/All	0.0000	0.0100	*8
	10.923	14.077	14.077	4.07	U1/All	0.0000	0.0000	
	14.077	17.230	17.230	12.22	U1/All	0.0000	0.0100	*8
	17.230	20.384	20.384	20.37	U1/All	0.0000	0.0100	*8
	20.384	24.750	23.537	28.52	U1/All	0.0097	0.0100	*8

2.8.3. Beam Transverse Reinforcement Details

Span	Size	Stirrups (2 legs each unless otherwise noted)
1	#3	16 @ 8.3 + <-- 37.8 --> + 16 @ 8.3

2.8.4. Beam Transverse Reinforcement Capacity

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

Span	Start	End	Required				Provided				
			X_u	V_u	Comb/Patt	A_v/s	A_v	Sp	A_v/s	ΦV_n	
	ft	ft	ft	kip		in²/in	in²	in	in²/in	kip	
1	0.000	0.250	1.463	28.52	U1/All	-----	-----	-----	-----	-----	
	0.250	10.923	1.463	28.52	U1/All	0.0097	0.22	8.3	0.0266	41.88	*8
	10.923	14.077	14.077	4.07	U1/All	0.0000	-----	-----	-----	10.42	
	14.077	24.750	23.537	28.52	U1/All	0.0097	0.22	8.3	0.0266	41.88	*8
	24.750	25.000	23.537	28.52	U1/All	-----	-----	-----	-----	-----	

2.9. Slab Shear Capacity

Span	b	d	V_{ratio}	ΦV_c	V_u	X_u
	in	in		kip	kip	ft
1 --- Not checked ---						

2.10. Material TakeOff

2.10.1. Reinforcement in the Direction of Analysis

Top Bars	0.0 lb	<=>	0.00 lb/ft	<=>	0.000 lb/ft²
Bottom Bars	255.0 lb	<=>	10.20 lb/ft	<=>	10.200 lb/ft²
Stirrups	52.1 lb	<=>	2.09 lb/ft	<=>	2.086 lb/ft²
Total Steel	307.1 lb	<=>	12.29 lb/ft	<=>	12.286 lb/ft²
Concrete	41.7 ft³	<=>	1.67 ft³/ft	<=>	1.667 ft³/ft²

3. Deflection Results: Summary

3.1. Section Properties

3.1.1. Frame Section Properties

Notes:
M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Span	Zone	M_{+ve}			M_{-ve}		
		I_g	I_{cr}	M_{cr}	I_g	I_{cr}	M_{cr}
		in⁴	in⁴	k-ft	in⁴	in⁴	k-ft
1	Left	8000	3756	32.98	8000	0	-32.98
	Midspan	8000	3756	32.98	8000	0	-32.98
	Right	8000	3756	32.98	8000	0	-32.98

3.1.2. Frame Effective Section Properties

Span	Zone	Weight	Load Level					
			Dead		Sustained		Dead+Live	
			M_{max}	I_e	M_{max}	I_e	M_{max}	I_e
			k-ft	in⁴	k-ft	in⁴	k-ft	in⁴
1	Middle	1.000	64.06	4335	64.06	4335	142.19	3809
	Span Avg	----	----	4335	----	4335	----	3809

3.2. Instantaneous Deflections

3.2.1. Extreme Instantaneous Frame Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	in	0.416	---	0.634	0.634	0.416	1.050

					Live			Total	
Span	Direction	Value	Units	Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
		Loc	ft	12.500	---	12.500	12.500	12.500	12.500
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---

3.3. Long-term Deflections

3.3.1. Long-term Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

		M_{+ve}					M_{-ve}				
Span	Zone	$A_{s,top}$	b	d	Rho'	Lambda	$A_{s,bot}$	b	d	Rho'	Lambda
		in²	in	in	%		in²	in	in	%	
1	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000

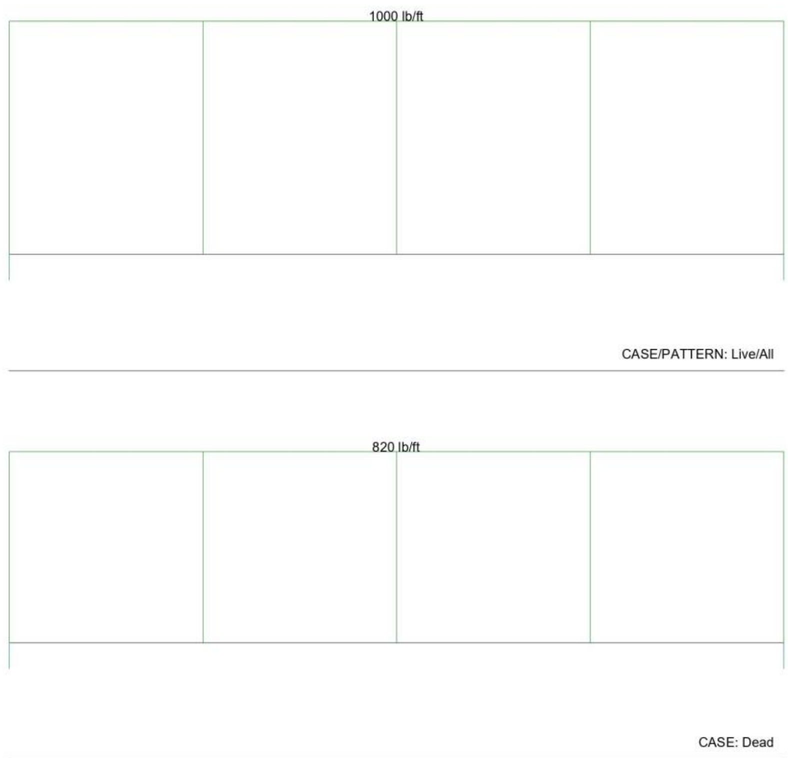
3.3.2. Extreme Long-term Frame Deflections and Corresponding Locations

Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	in	0.831	1.466	1.466	1.881
		Loc	ft	12.500	12.500	12.500	12.500
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---

4. Diagrams

4.1. Loads



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Frame: Simply Supported RC Beam
Engineer: SP
Code: ACI 318-14
Date: 04/08/21
Time: 10:41:11

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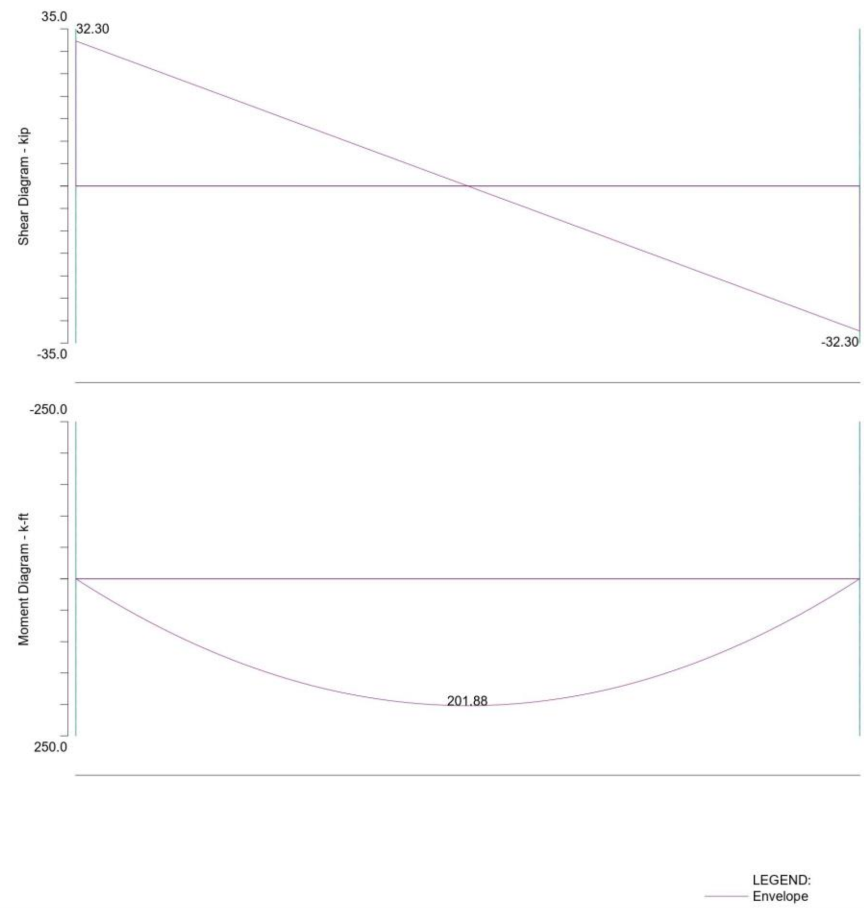
4.2. Internal Forces



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Frame: Simply Supported RC Beam

Engineer: SP

Code: ACI 318-14

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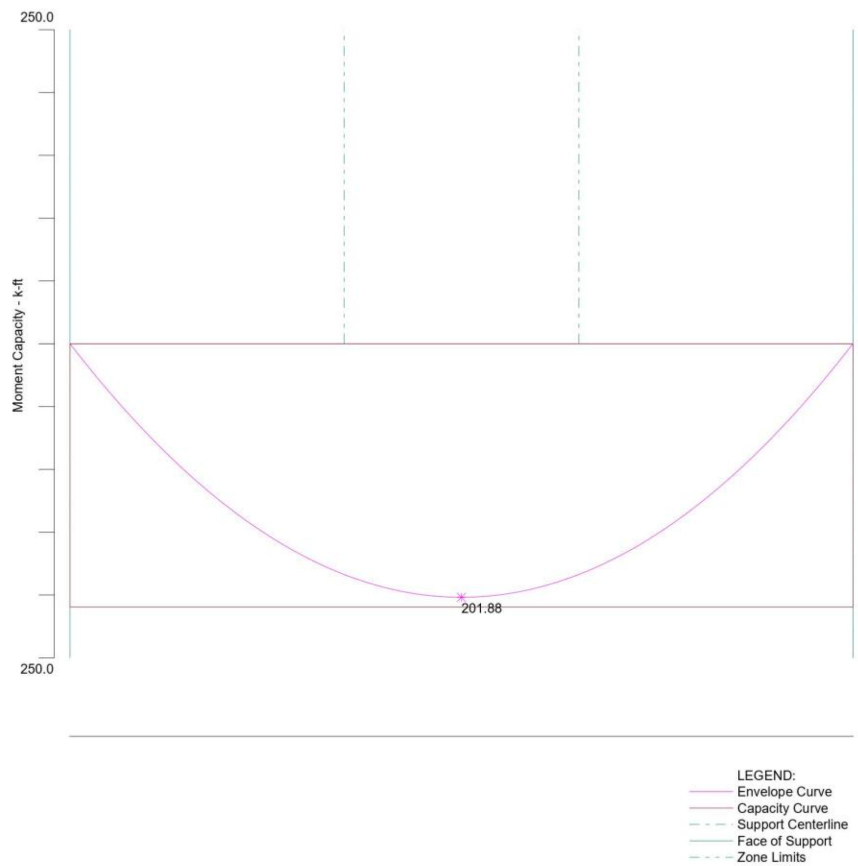
4.3. Moment Capacity



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Engineer: SP

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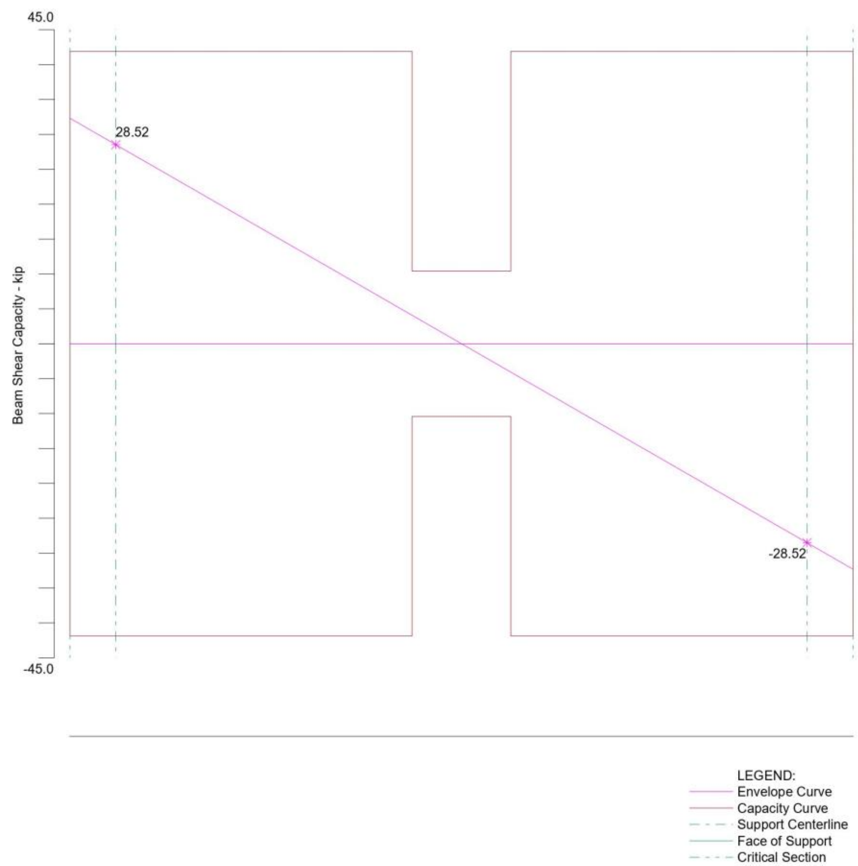
4.4. Shear Capacity



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Frame: Simply Supported RC Beam

Engineer: SP

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Date: 04/08/21

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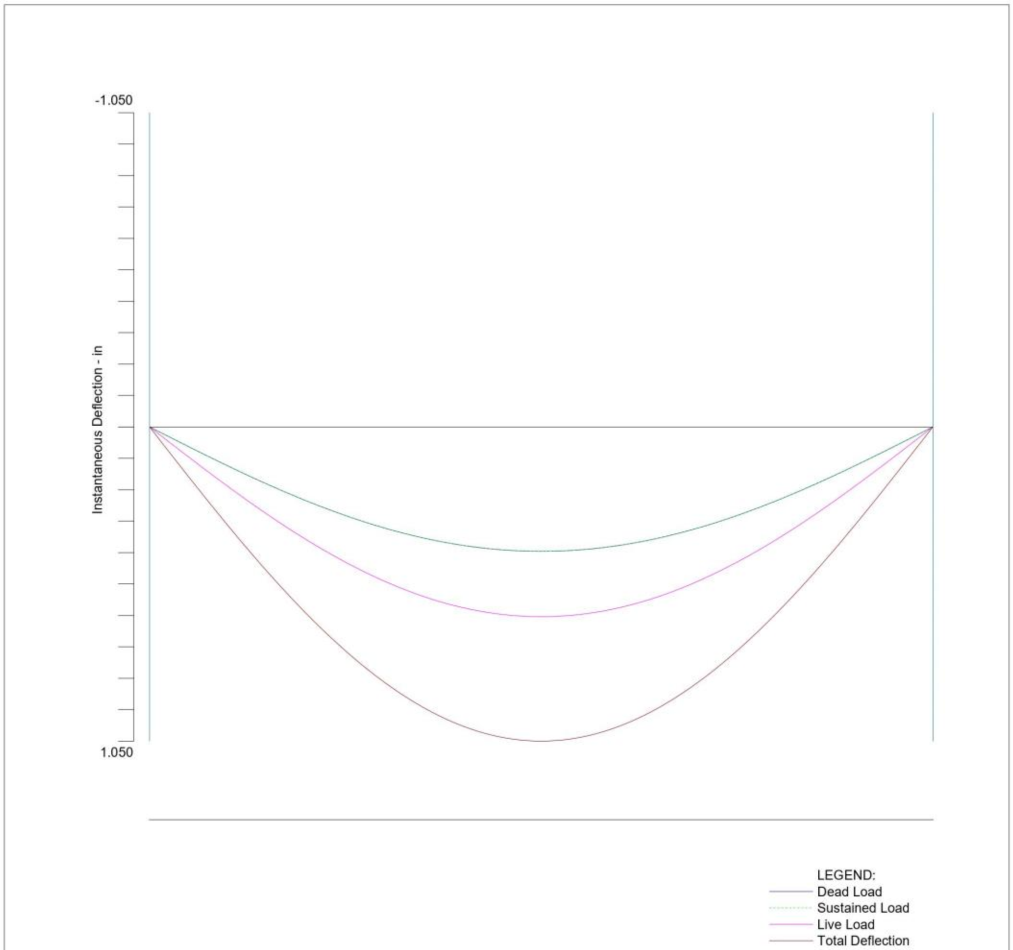
4.5. Deflection



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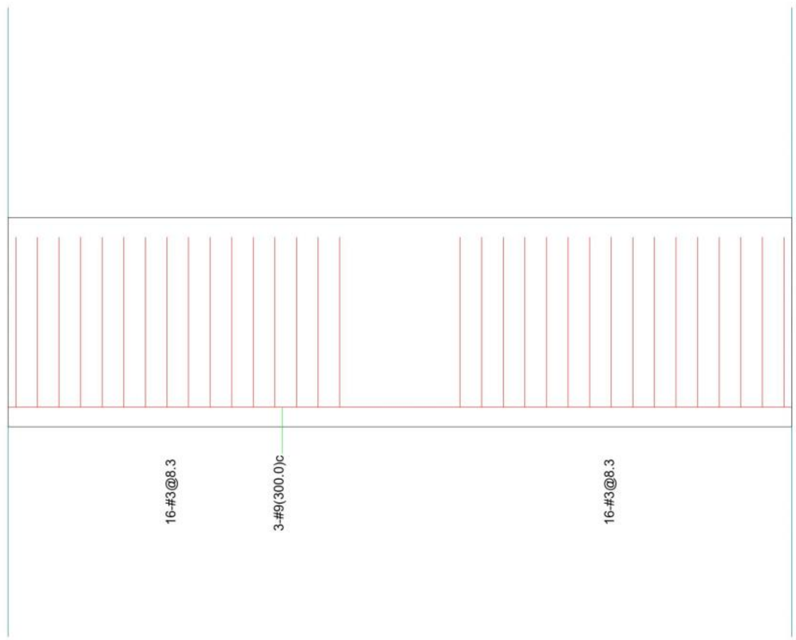
4.6. Reinforcement



Flexural and Transverse Reinforcement

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Frame: Simply Supported RC Beam
Engineer: SP
Code: ACI 318-14
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8. Analysis and Design Results Comparison and Conclusions

The following tables show the comparison between hand results and spBeam model results.

Table 2 - Comparison of Moments and Flexural Reinforcement (At Midspan)				
Location	M_u, kip-ft	$A_{s,required}$, in.^{2}	Reinforcement	$A_{s,provided}$, in.^{2}
Hand	201.88	2.872	3 – #9	3.000
spBeam	201.88	2.873	3 – #9	3.000

Table 3 - Comparison of Shear and lateral Reinforcement									
$V_{u@d}$, kip		$(A_v/s)_{req}{}^*$, in.^{2}/ in.		$(A_v/s)_{min}{}^*$, in.^{2}/ in.		Reinforcement		ϕV_n, kip	
Hand	spBeam	Hand	spBeam	Hand	spBeam	Hand	spBeam	Hand	spBeam
28.52	28.52	0.0097	0.0097	0.0100	0.0100	#3 @ 8.3 in.	#3 @ 8.3 in.	41.79	41.88
* Minimum transverse reinforcement governs									

Table 4 - Comparison of Section Properties								
Location	I_{cr}, in.^{4}		I_e, in.^{4}					
	Hand	spBeam	Hand			spBeam		
			DL	DL+LL_{sus}	Total	DL	DL+LL_{sus}	Total
Midspan	3759	3756	4337	4337	3812	4335	4335	3809

Table 5 - Comparison of Maximum Instantaneous Deflection (At Midspan), in.		
Deflection Type	Hand	spBeam
Δ_{DL}	0.416	0.416
Δ_{LL}	0.634	0.634
Δ_{total}	1.050	1.050

Table 6 - Comparison of Maximum Long-Term Deflection (At Midspan), in.		
Deflection Type	Hand	spBeam
Δ_{cs}	0.831	0.831
$\Delta_{cs} + \Delta_{LL}$	1.465	1.466
$(\Delta_{total})_{lt}$	1.881	1.881

The results of all the hand calculations used illustrated above are in agreement with the automated exact results obtained from the spBeam program.